COSC326  
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Etude 13: Missing Scores  
  
Q1)  
From the information given:  
Aggregate scores are all different.  
There were no ties.

Alan had a score of 24(IE, maximum in all but 1)  
Charles had the same mark in 4 papers.  
Ellen had full marks for mathematics, and 3 marks in Science.  
The aggregate marks is from best to worst, in alphabetical order.

From this, we can gather than Alan’s mark of 4 is in Mathematics, since Ellen took the top score there:  
IE:

In order from left to right:   
Name, English, History, French, Mathematics, Science, Aggregate Score

A 5 5 5 4 5  
B x x x x x  
C x x x x x  
D x x x x x  
E x x x 5 3

The next step we look at is the smallest possible minimum for aggregate range remaining, which if E has all 1’s as a value, would be 75(The total possible aggregate) – 11 – 24(the potential known aggregate), which would leave us with a potential 40. We can see if this aggregate is well divided between B, C and D, where 11 < D < C < B < 24. This is an important part for later.

Our next step now is to find out what C’s four values could potentially be.  
If we assume 4C’s = 4(ie, 4 of them have a value of 1), it would be less than the known potential aggregate and even smallest possible value. Hence, it cannot be 1. If 4C’s = 8, we can consider that E is essentially 8 + ?, where ? represents all possible additional values to the aggregate. To demonstrate:

C: 2, 2, 2, 2, (1, 2, 4) – This is because we already know that it cannot be 3 or 5 thanks to the previous information.

E: (1-4), (1-4), (1-4), 5, 3 – Potential ranges

Note that the smallest possible value for E is 11, meaning that only 4C + 4 is viable. But, as D is in-between E and C, this implies that D will equal either E or C if 4C = 8. Thus, it cannot be 2 either.

To eliminate the next highest value, we will look at 4 next. Consider 4C = 16 + (1-3). Is it possible for B to get a higher value than this after eliminating 4 from the equation? It is quite impossible, as even the next possible value, 3, would at maximum give 12, and all 4’s would at that point have been taken. Thus, the only possible combination is now 3.

IE:

In order from left to right:   
Name, English, History, French, Mathematics, Science, Aggregate Score

A 5 5 5 4 5  
B x x x x x  
C 3 3 3 3 (1,2,4) – Note that because E has 3 at the last slot, only that value can be changed at this point.  
D x x x x x  
E x x x 5 3

At this point, we can consider the variant ranges we have in both C and E. If we look at C, its maximum possible aggregate is 16, and minimum is 13. Whereas for E, the minimum is 11 and maximum is 3E = 12 + 5 + 3 = 20. We can immediately eliminate this, and any value that exceeds 11. This is because C’s most base known aggregate at this point is 12, meaning anything below that is viable. If we take the range values, we can increase the variables, but at most they can never exceed 16-2, as that is the biggest C can be, and – 2 as E cannot equal D, and D cannot equal C. therefore, anything exceeding 14 in value is thrown away as impossible. This leaves the possible ranges of 1, (1-2), (1,2,4).

Now let us consider aggregate sums where ranges apply. Assume 5C = 13, IE, 4\*3 + 1. This would imply that our possible ranges for E are:   
1, 1, 1 = 3  
1, 1, 2 = 4  
1, 2, 2 = 5  
1, 1, 4 = 6  
1, 2, 4 = 7 - Impossible due to exceeding 14(7 + 8)

As we are only interested in aggregate sums, I left out similar values.

Now, we can see that with C = 13, E = 11, and any exceeding values are thrown away. Just to confirm this, consider C = 14, E = 11, 12. Let us look at the potential aggregates at this point, where 14 + 11 + 24 = 49, leaving 26, or 25 in the case of E = 12. If we wish to split 26 for B and D:  
11 < D < 14  
14 < B < 24  
  
Therefore, D can only be 12 or 13. B = 26 – 12 = 14 leads us to an impossible case, as B cannot be equal to C. In the case of 13, B cannot be less than 14. Looking at the case where E = 12:  
12 < D < 14  
14 < B < 24

D = 13, but with B = 25 – 13 being 12, and 12 > 14, it implies another impossibility. Therefore, the only possible case is C = 13, which shows that the last part must be 1.

IE:

In order from left to right:   
Name, English, History, French, Mathematics, Science, Aggregate Score

A 5 5 5 4 5, 24  
B x x x x x, ?  
C 3 3 3 3 1, 13  
D x x x x x, ?  
E x x x 5 3, 8 + ?

With the above, we can gather that whatever is added for E cannot be 12 or 13 due to C already taking 13, therefore its values can only be 1. Furthermore, we can now get a total aggregate from 24 + 13 + 11, which shows 24 + 24 = 48, taking the total remaining possible aggregate score 75 – 48 = 27. This implies that the aggregate score at D must be 12, and by proxy B must be 15. We can confirm this from plugging in arbitrary values that total the possible aggregate score.

IE:

In order from left to right:   
Name, English, History, French, Mathematics, Science, Aggregate Score

A 5 5 5 4 5, 24  
B x x x x x, 15  
C 3 3 3 3 1, 13  
D x x x x x, 12  
E 1 1 1 5 3, 11

We know that B > D and can only be 4 or 2, except for B which can only be 2 or 1.  
We have the remaining values: one 1, four 4’s, and five 2’s.  
To get 15, we can look at 3\*4 + 2 + 1 = 15  
To get 12, we can see if the remaining totals up. 4 + 4\*2 = 12  
This checks out, but we need to know where to place the values. If we look carefully, we can see that we have only one place for 1 to be put. This shows the following table:

IE:

In order from left to right:   
Name, English, History, French, Mathematics, Science, Aggregate Score

A 5 5 5 4 5, 24  
B x x x 1 x, 15  
C 3 3 3 3 1, 13  
D x x x 2 x, 12  
E 1 1 1 5 3, 11  
Any remaining positions(x) in B can represent 3 \* 4 and 2.  
The remaining positions(x) in D can represent 4 and 3 \* 2.

We can also note that E was the proper minimum, but the elimination of any other possible method through elimination logic and potential range confirmed that this was the most viable potential solution.

Therefore, Barbara’s(B)’s mark in Mathematics is 1, and there are three students: Alan getting 5 in four subjects, Charles getting 3 in four subjects and David getting 2 in four subjects.

Q2)